$$e = 3\,N_{C_2HCl_3}$$

$$f = N_{Al}$$

$$Hf = \sum_i N_i Hf_i$$

where

$N_i \equiv$ moles of component i/100 g of propellant
$Hf_i \equiv$ molar enthalpy of formation of component i

Given the enthalpies of formation of the components, the equations may be solved to find the composition of a mixture which will provide an exhaust indistinguishable from that of the solid propellant. The simulation will be valid only if equilibrium combustion occurs in both cases. A further limitation is that in the case of multiphase exhausts the particle size distribution will not, necessarily, be duplicated.

In practice, a proposed composition may for some reason prove undesirable. In the case cited, for example, the slurry required might be too concentrated for practical use, or there might be ignition problems. In such an event, a liquid aluminum derivative might be used or the gaseous oxygen might be replaced by a liquid oxidizer. The only requirement is that an element and enthalpy balance be obtained, and the limitation is thus only with regard to the number of suitable compounds of known composition and enthalpy of formation which are available.

Exact simulation may not always be necessary or practical. Formulation might be greatly simplified by omission of one of the minor components, with little effect on the exhaust characteristics. Also, systematic variation of the exhaust temperature and chemistry might be desired. In the foregoing example, for instance, the component $C_2H_8N_2(l)$ and the heat balance equation might be omitted in the calculation. The nitrogenous compounds in the exhaust would then arise entirely from $N_2(g)$. The atomic composition of the exhaust would be the same as before, but the temperature and chemical composition would be different.

Thermoelastic Vibrations of Heterogeneous Membranes and Inextensional Plates

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IN Ref. 1 a general nonlinear thermoelastic theory was established for thin heterogeneous aeolotropic plates. In the following, the author specializes the general equations to the cases of 1) membranes, 2) pseudo-membranes, and 3) inextensional plates, including thermally induced transverse vibrations.

The two simultaneous nonlinear equations for w and F established previously[1] are

$$L_1w - L_3F = p + L_BN + L_tM + K(F,w) \tag{1}$$

$$L_2F + L_3w = L_AN - \tfrac{1}{2}K(w,w) \tag{2}$$

where

$$L_AN = L_{Ax}\,N_{xt} + L_{Ay}N_{yt} + L_{As}N_{st} \tag{3}$$

$$L_BN = L_{Bx}N_{xt} + L_{By}N_{yt} + L_{Bs}N_{st} \tag{4}$$

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$$L_tM = M_{xt,xx} + 2M_{st,xy} + M_{yt,yy} \tag{5}$$

$$K(F,w) = F_{,yy}w_{,xx} - 2F_{,xy}w_{,xy} + F_{,xx}w_{,yy} \tag{6}$$

$$\tfrac{1}{2}K(w,w) = w_{,xx}w_{,yy} - w_{,xy}{}^2 \tag{7}$$

and the other operators are given in Ref. 1.

Note that the linear coupling operator L_3 vanishes for homogeneous plates and for some special classes of heterogeneity,[2] e.g., for symmetrically laminated plates. For such systems a membrane state exists whenever $L_1 \rightarrow 0$, and an inextensional state exists whenever $L_2 \rightarrow 0$. However, for a general heterogeneous system characterized by Eqs. (1) and (2), one distinguishes between a pseudo-membrane state where $L_1 = 0$ and a classical membrane state where, in addition to L_1, the operator L_3 also vanishes.

The finite deflection equations of a heterogeneous aeolotropic pseudo-membrane, which also undergoes thermal gradients, are as follows:

$$-L_3F = p + L_BN + L_tM + K(F,w) \tag{8}$$

$$L_2F + L_3w = L_AN - \tfrac{1}{2}K(w,w) \tag{9}$$

It is noted that the pseudo-membrane state is not free of bending moments, as can be seen from Eqs. (29) of Ref. 1. One also may distinguish between a pseudo-inextensional state, where $L_2 = 0$ in Eqs. (1) and (2), and a classical inextensional state, where L_3, as well as L_2, vanish.

The equations of a pseudo-inextensional heterogeneous plate take the form

$$L_1w - L_3F = p + L_BN + L_tM + K\,(F,w) \tag{10}$$

$$L_3w = L_AN - \tfrac{1}{2}K(w,w) \tag{11}$$

Note that the assumption of pseudo-inextensibility reduces the coupled eighth-order system of simultaneous Eqs. (1) and (2) to two successive fourth-order equations. The coupling of w and F will, however, still be retained in the boundary conditions, as can be seen from Eqs. (10) and (19) of Ref. 2.

So far the effect of inertia was disregarded in the analyses; however, in problems of thermally induced vibrations of plates, it must be considered. To account for transverse vibrations of the heterogeneous systems, apply Eqs. (8) and (9) or (10) and (11) as the case is, except that the lateral load p is replaced by an inertia load term of the form $-R_0w_{,tt}$, where

$$R_0 = \int_0^h \rho\, dz \tag{12}$$

and $\rho = \rho(z)$ is the density of the plate material.

Accordingly, the differential equations for free pseudo-membrane lateral vibrations or free inextensional transverse vibrations are Eqs. (8) and (9) or Eqs. (10) and (11), respectively, where now

$$w = w(x,y,t) \tag{13}$$

$$F = F(x,y,t) \tag{14}$$

and neglecting longitudinal inertia

$$p = p(x,y,t) = -R_0w_{,tt} \tag{15}$$

Equations (8) and (9) include, as a special case, Föppl's[3] equations for finite deflections of homogeneous membranes.

Equations (10) and (11) with (13–15) include, as a special case, Boley and Barber's[4] equation for thermally induced vibrations of homogeneous plates.

The author finally mentions a particular case of deflection of a pseudo-membrane to a cylindrical surface whose axis is parallel to the y axis.

Considering uniform heating, Eqs. (8) and (9) are simplified to

$$B_{yx}{}^*F_{,xxxx} = p + F_{,yy}w_{,xx} \tag{16}$$

$$A_{yy}{}^*F_{,xxxx} - B_{yx}{}^*w_{,xxxx} = 0 \quad (17)$$

where

$$F_{,yy} = N_0 = \text{const} \quad (18)$$

and $A_{yy}{}^*$, $B_{yx}{}^*$ are given by Eqs. (18) of Ref. 2.

Reducing Eqs. (16) and (17) to a single equation in terms of w, one gets

$$(A_{yy}{}^*)^{-1}(B_{yx}{}^*)^2\, w_{,xxxx} = p + N_0 w_{,xx} \quad (19)$$

which resembles Eq. (a) on p. 7 of Ref. 5 for finite cylindrical bending of homogeneous plates.

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Comments

Comment on "Characteristics of the Arc in a Gerdien-Type Plasma Generator"

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In a recent technical note,[1] the authors have shown that in a Gerdien-type plasma generator exhausting into the atmosphere, where argon is the working fluid, the arc process is cyclic, and the observed flame consists of the envelope of many blown arc current paths moving about at high frequency. Measurements using equilibrium radiation concepts and based on the assumption of an ionized, current-free gas stream would be in error when applied to this stream. To provide a source of ionized gas free of current paths, the stream was expanded into a low pressure test cell (≈ 0.5 mm Hg) and the expanded flame carefully examined to determine whether the stream was influenced by the same blown arc phenomena. The same type of probe experiments as for the atmospheric case were performed.

The expanded luminous gas stream is shown in Fig. 1. The very bright section at the orifice exit contains the many blown arcs and is about the same size as the radiating jet at atmospheric pressure. The expanded flame does not contain the current paths, and electron velocity distributions found in the expanded region by Langmuir probes were Maxwellian. The expanded flame should, therefore, provide an acceptable source for further study of measurement techniques.

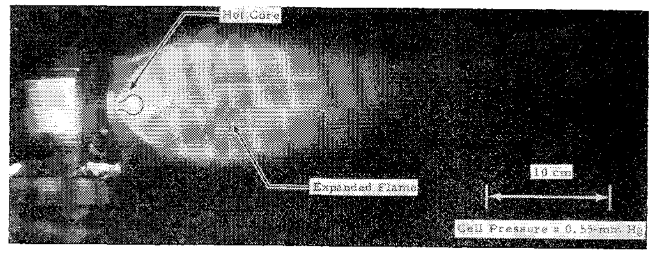

Fig. 1 Plasma generator exhausting to low pressure, showing hot core blown arc region and current free expanded flame

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Comments on "Treatment of Partial Equilibrium in Chemically Reacting Flow"

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Libby[1] has presented a method for dealing with chemically reacting flows when approximate local equilibrium is maintained among some but not all of the chemical species present. The purpose of the present comments is to point out that the solution presented may be indeterminate in some cases.

The difficulty lies in the fact that the G_i are not all independent. The relationship between the G_i is most clearly seen if the N-L independent G_j are chosen as those for the reactions by which species 1, . . ., N-L are formed from the elements. This set of G_j must be independent, since each contains only one of the Y_i, $i = 1, \ldots, N\text{-}L$. It then can be shown that

$$1 - G_i = \prod_{j=1}^{N-L} (1 - G_j)^{\nu_{ij}' - \nu_{ij}''} \quad i = N\text{-}L + 1, \ldots, K \quad (1)$$

The σ_j, which are functions of the rate constants, are all independent. Thus, in general, Eqs. (6) in footnote 1 will not all be linearly independent, and the M indeterminate σ_j cannot be found. This problem will not occur in all cases. If the reaction mechanism involves only N-L reactions, all of the G_j will be independent. Similarly if the species that are assumed to be in local equilibrium are each involved in only one reaction, the system of Eqs. (6) will be linearly independent. However, most real chemical mechanisms, particularly at high temperatures, involve many more than N-L reactions, and in these cases it is unlikely that a solution can be found.

Physically, this problem arises because once equilibrium is assumed it is no longer meaningful to compute contributions to gradients in species concentrations due to individual reactions. Once a species is assumed to be in local chemical equilibrium with some other species, its concentration can be

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